

# Holographic Dual of Linear Dilaton Black Hole in Einstein-Maxwell-Dilaton-Axion Gravity

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**ABSTRACT:** Motivated by the recently proposed Kerr/CFT correspondence, we investigate the holographic dual of the extremal and non-extremal rotating linear dilaton black hole in Einstein-Maxwell-Dilaton-Axion Gravity. For the case of extremal black hole, by imposing the appropriate boundary condition at spatial infinity of the near horizon extremal geometry, the Virasoro algebra of conserved charges associated with the asymptotic symmetry group is obtained. It is shown that the microscopic entropy of the dual conformal field given by Cardy formula exactly agrees with Bekenstein-Hawking entropy of extremal black hole. Then, by rewriting the wave equation of massless scalar field with sufficient low energy as the  $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$  Casimir operator, we find the hidden conformal symmetry of the non-extremal linear dilaton black hole, which implies that the non-extremal rotating linear dilaton black hole is holographically dual to a two dimensional conformal field theory with the non-zero left and right temperatures. Furthermore, it is shown that the entropy of non-extremal black hole can be reproduced by using Cardy formula.

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## 1. Introduction

The recently proposed extremal Kerr/CFT correspondence [1] states that quantum gravity in the region very near the event horizon of an extreme Kerr black hole with proper boundary conditions is holographically dual to a two-dimensional chiral conformal field theory with the central charge proportional to angular momentum. The method employed by Guica, Hartman, Song and Strominger (GHSS) in [1] is very similar to the approach of Brown and Henneaux in [2], where the  $\text{AdS}_3$  background is replaced by the near-horizon extremal Kerr (NHEK) geometry previously obtained in [3]. They shown that, by imposing the appropriate boundary condition at spatial infinity of the NHEK geometry, the conserved charges associated with the asymptotic symmetry group are found to form a copy of Virasoro algebra. If identifying this algebra with the Virasoro algebra of the dual two dimensional conformal field theory, it is shown that the macroscopic Bekenstein-Hawking entropy of extremal Kerr black hole can be reproduced by the microscopic entropy of dual conformal field theory via Cardy formula. This method has been generalized to calculate the entropies of extremal black holes in a lot of theories such as the Einstein theory, string theory, and supergravity theory. Some further studies on the extremal Kerr/CFT dual are listed in [5]-[40].

More recently, Castro, Maloney and Strominger (CMS) in a remarkable paper [41] show that there exists a hidden  $\text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R$  conformal symmetry for the four dimensional non-extremal Kerr black hole by studying the near-region wave equation of a massless scalar field. Interestingly, this hidden conformal symmetry is not derived from the conformal symmetry of spacetime geometry itself, but probed by the perturbation fields in the near region. It is shown that, for the massive scalar field in the background of Kerr black hole with sufficient low energy, the wave equation can be reproduced by the  $\text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R$  Casimir operator. It is also shown that microscopic entropy computed by Cardy formula agrees exactly with the macroscopic Berenstein-Hawking entropy of non-extremal Kerr black hole. These observations suggest that non-extremal Kerr black hole

is also holographically dual to a two-dimensional conformal field theory with non-zero left and right temperatures. Some related works on the hidden conformal symmetry of non-extremal black holes are listed in [42]-[54].

Among the works on the Kerr/CFT dual, the asymptotic geometries of background spacetimes are of the flat or AdS type. In the present paper, we consider a rotating black hole in Einstein-Maxwell-Dilaton-Axion (EMDA) Gravity which is asymptotic to the linear dilaton spacetime. By using the approaches of Kerr/CFT dual, the holographic dual of the extremal and non-extremal rotating linear dilaton black hole is investigated. For the case of extremal black hole, the NHEK geometry is found after performing a coordinates transformation. This geometry is of the  $U(1)_L \times SL(2, R)_R$  symmetry. By imposing the appropriate boundary condition at spatial infinity of the NHEK geometry, it is shown that the  $U(1)_L$  symmetry is enhanced into a Virasoro algebra with the central charge  $c = 12J$ , where  $J$  is the angular momentum. One can conjecture that there exists a dual conformal field theory for the extremal linear dilaton black hole by identifying the Virasoro algebra with that of CFT. It is shown that the microscopic entropy of the dual conformal field given by Cardy formula exactly agrees with Bekenstein-Hawking entropy of extremal black hole.

Then, by studying the wave equation of massless scalar field in this background, we study the hidden  $SL(2, R)_L \times SL(2, R)_R$  conformal symmetry of the non-extremal black hole. We firstly find that the radial equation can be exactly solved by the hypergeometric functions. As hypergeometric functions transform in representations of  $SL(2, R)$ , this suggests the existence of a hidden conformal symmetry. Furthermore, it is explicitly shown that the wave equation of scalar field can also be obtained by using of the  $SL(2, R)_L \times SL(2, R)_R$  Casimir operator, which implies that the non-extremal rotating linear dilaton black hole is holographically dual to a two dimensional conformal field theory with the non-zero left and right temperatures. As a check of this conjecture, we also show that the entropy of non-extremal linear dilaton black hole can be reproduced by using Cardy formula.

This paper is organized as follows. In section II, we give a brief review of rotating linear dilaton black hole in Einstein-Maxwell-Dilaton-Axion Gravity. In section III, we obtain the near horizon geometry of extremal black hole and calculate the central charge and the left and right temperatures of the dual conformal field theory. We also find Bekenstein-Hawking entropy of extremal black hole matches with the microscopic entropy of dual CFT. In section IV, we study the hidden conformal symmetry of the non-extremal linear dilaton black hole by analysing the near-region wave equation of massless scalar field. Furthermore, the microscopic entropy of dual CFT with non-zero left and right temperatures are obtained. The last section is devoted to conclusion and discussion.

## 2. Rotating linear dilaton black hole

In this section, we will give a brief review about a class of rotating black hole solution of Einstein-Maxwell-dilaton-axion gravity in four dimensions with the linear dilaton background reported by Clément et al in [55]. The EMDA gravity theory can be considered arising as a truncated version of the bosonic sector of  $D = 4$ ,  $N = 4$  supergravity theory.

The action of EMDA gravity theory is given by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ -R + 2\partial_\mu \phi \partial^\mu \phi + \frac{1}{2} e^{4\phi} \partial_\mu \kappa \partial^\mu \kappa - e^{-2\phi} F_{\mu\nu} F^{\mu\nu} - \kappa F_{\mu\nu} \tilde{F}^{\mu\nu} \right], \quad (2.1)$$

where  $\phi$  and  $\kappa$  are the dilaton field and the axion (pseudoscalar) field respectively,  $F$  and  $\tilde{F}$  are the field strength of abelian vector field  $A$  and its dual. Using the solution generating technique, the rotating black hole solution is obtained in [55], where the metric without Nut charge is explicitly given by

$$ds^2 = -\frac{\Delta}{r_0 r} dt^2 + r_0 r \left[ \frac{dr^2}{\Delta} + d\theta^2 + \sin^2 \theta \left( d\varphi - \frac{a}{r_0 r} dt \right)^2 \right], \quad (2.2)$$

where the factor  $\Delta = (r^2 - 2Mr + a^2)$ , and the other background fields are given by

$$\begin{aligned} F &= \frac{1}{\sqrt{2}} \left[ \frac{r^2 - a^2 \cos^2 \theta}{r_0 r^2} dr \wedge dt + a \sin 2\theta d\theta \wedge \left( d\varphi - \frac{a}{r_0 r} dt \right) \right], \\ e^{-2\phi} &= \frac{r_0 r}{r^2 + a^2 \cos^2 \theta}, \\ \kappa &= -\frac{r_0 a \cos \theta}{r^2 + a^2 \cos^2 \theta}. \end{aligned} \quad (2.3)$$

The metric (2.2) was derived from the Kerr metric by using the solution generating technique. When  $r \rightarrow \infty$ , the rotating linear dilaton black hole is asymptotic to the linear dilaton background, which is different with that of Kerr black hole.

We now summarize the thermodynamics of rotating linear dilaton black hole. It should be noted that  $M$  appeared in the solution is no longer the physical mass. In order to obtain the first law of black hole mechanics, the relevant thermodynamics quantities can be calculated by using the approach of Brown and York [56]. The mass  $\widetilde{M}$  of rotating linear dilaton black hole is relative to the mass parameter  $M$  towards the formula

$$\widetilde{M} = \frac{M}{2}, \quad (2.4)$$

and the angular momentum  $J$  is given by

$$J = \frac{a r_0}{2}. \quad (2.5)$$

The Hawking temperature  $T_H$ , the Bekenstein-Hawking entropy  $S_{BH}$  and the angular velocity  $\Omega_H$  of the event horizon are given as

$$\begin{aligned} T_H &= \frac{r_+ - r_-}{4\pi r_0 r_+}, \\ \Omega_H &= \frac{a}{r_0 r_+}, \\ S_{BH} &= \pi r_0 r_+, \end{aligned} \quad (2.6)$$

where  $r_\pm = M \pm \sqrt{M^2 - a^2}$  are the locations of the outer and the inner event horizon respectively. With the thermodynamical quantities given above, one can deduce the differential first law of black hole mechanics by straightforward calculation

$$d\widetilde{M} = T_H dS_{BH} + \Omega_H dJ. \quad (2.7)$$

It should be noted that the electric charge  $Q = r_0/\sqrt{2}$  does not appear in the above thermodynamics relation. The differentiations are performed keeping  $Q$  as a fixed value, which is characteristic of the linear dilaton background. When employing the approach of Brown and York to calculate the conserved quantities of the non-asymptotically flat spacetime, one should make a choice of the background 'vacuum' solution. In the present case, the electric charge  $Q$  is selected to be the background charge. One can refer to [55] for more details.

The extremality condition is  $M = a$ , and the entropy at extremality is

$$S_{BH}(T_H = 0) = \pi r_0 a . \quad (2.8)$$

In the following two sections, we will try to reproduce the Bekenstein-Hawking entropies of the extremal and non-extremal linear dilaton black holes by using Cardy formula of the dual conformal field.

### 3. Holographic dual of extremal linear dilaton black hole

In this section, our purpose is to derive the Virasoro algebra of the dual conformal field by studying the asymptotic symmetry group of near horizon extremal geometry of linear dilaton black hole and reproduce its Bekenstein-Hawking entropy via Cardy formula.

Firstly, we now try to explore the near-horizon geometry of extremal linear dilaton black hole. To do so, we need to perform the following coordinate transformations

$$\begin{aligned} r &= a + \epsilon \lambda \hat{r} , \\ t &= \frac{\lambda \hat{t}}{\epsilon} , \\ \varphi &= \hat{\varphi} + \frac{1}{r_0} \frac{\lambda \hat{t}}{\epsilon} , \end{aligned} \quad (3.1)$$

with the parameter  $\lambda^2 = r_0 a$ . After taking the  $\epsilon \rightarrow 0$  limit, one can obtain the near-horizon geometry for an extremal rotating linear dilaton black hole

$$ds^2 = r_0 a \left( -\hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + d\theta^2 \right) + r_0 a \sin^2 \theta (d\hat{\varphi} + \hat{r} d\hat{t})^2 , \quad (3.2)$$

and the near-horizon gauge field, dilaton field and axion field

$$\begin{aligned} F &= \frac{a}{\sqrt{2}} \left[ \sin^2 \theta d\hat{r} \wedge d\hat{t} + \sin 2\theta d\theta \wedge (\hat{\varphi} + \hat{r} d\hat{t}) \right] , \\ e^{-2\phi} &= \frac{r_0}{a(1 + \cos^2 \theta)} , \\ \kappa &= -\frac{r_0 \cos \theta}{a(1 + \cos^2 \theta)} . \end{aligned} \quad (3.3)$$

It is worth noting that, although the asymptotic behavior of linear dilaton black hole is different with that of Kerr black hole, the near-horizon metric (3.2) and the near-horizon background fields of an extremal linear dilaton black hole take the same form as that of an extremal Kerr black hole.

The NHEK geometry (3.2) has an enhanced  $U(1)_L \times SL(2, R)_R$  isometry group, which are respectively generated by the Killing vectors

$$K_1 = \partial_{\hat{\varphi}} , \quad (3.4)$$

and

$$\begin{aligned} \bar{K}_1 &= \partial_{\hat{t}} , \\ \bar{K}_2 &= \hat{t}\partial_{\hat{t}} - \hat{r}\partial_{\hat{r}} , \\ \bar{K}_3 &= \left( \frac{1}{2\hat{r}^2} + \frac{\hat{t}^2}{2} \right) \partial_{\hat{t}} - \hat{t}\hat{r}\partial_{\hat{r}} - \frac{1}{\hat{r}}\partial_{\hat{\varphi}} . \end{aligned} \quad (3.5)$$

We now employ the approach of Brown and Henneaux to find the central charge of the holographic dual conformal field theory description of an extremal rotating linear dilaton black hole. Because the linear dilaton black hole is a solution of EMDA gravity theory, it seems that there exists the non-vanishing contributions to the central charge from gauge field, dilaton field and axion field. Fortunately, an explicit calculation given by Compere et al. in [6] shows that, in Einstein-Maxwell-Dilaton theory with topological terms in four and five dimensions, the central charge receives no contribution from the non-gravitational fields, i.e. only the Einstein-Hilbert Lagrangian contributes to the value of the central charge. To find the central charge of the dual conformal field for the rotating linear dilaton black hole, for simplicity, it is sufficient to only calculate the gravitational field contribution.

It is important to impose the appropriate boundary conditions at spatial infinity of NHEK geometry (3.2) and find the asymptotical symmetry group that preserves these boundary conditions. For the metric fluctuations around the NHEK geometry, we impose the boundary conditions

$$\begin{pmatrix} h_{\hat{t}\hat{t}} = \mathcal{O}(\hat{r}^2) & h_{\hat{t}\hat{\varphi}} = \mathcal{O}(1) & h_{\hat{t}\hat{\theta}} = \mathcal{O}(\frac{1}{\hat{r}}) & h_{\hat{t}\hat{r}} = \mathcal{O}(\frac{1}{\hat{r}^2}) \\ h_{\hat{\varphi}\hat{\varphi}} = \mathcal{O}(1) & h_{\hat{\varphi}\hat{\theta}} = \mathcal{O}(\frac{1}{\hat{r}}) & h_{\hat{\varphi}\hat{r}} = \mathcal{O}(\frac{1}{\hat{r}}) \\ h_{\hat{\theta}\hat{\theta}} = \mathcal{O}(\frac{1}{\hat{r}}) & h_{\hat{\theta}\hat{r}} = \mathcal{O}(\frac{1}{\hat{r}^2}) \\ h_{\hat{r}\hat{r}} = \mathcal{O}(\frac{1}{\hat{r}^3}) \end{pmatrix} \quad (3.6)$$

where  $h_{\mu\nu}$  is the metric deviation from the near horizon geometry.

The most general diffeomorphism symmetry that preserves such a boundary condition is generated by the vector field

$$\zeta = \epsilon(\hat{\varphi}) \frac{\partial}{\partial \hat{\varphi}} - \hat{r} \epsilon'(\hat{\varphi}) \frac{\partial}{\partial \hat{r}} , \quad (3.7)$$

where  $\epsilon(\hat{\varphi})$  is an arbitrary smooth periodic function of the coordinate  $\hat{\varphi}$ . It is convenient to define  $\epsilon_n(\hat{\varphi}) = -e^{-in\hat{\varphi}}$  and  $\zeta_n = \zeta(\epsilon_n)$ , where  $n$  are integers. Then the asymptotic symmetry group is generated by

$$\zeta_n = -e^{-in\hat{\varphi}} \frac{\partial}{\partial \hat{\varphi}} - in\hat{r}e^{-in\hat{\varphi}} \frac{\partial}{\partial \hat{r}} , \quad (3.8)$$

which obey the Virasoro algebra with vanishing central charge

$$i[\zeta_m, \zeta_n] = (m - n)\zeta_{m+n} . \quad (3.9)$$

Each diffeomorphism  $\zeta_n$  is associated to a conserved charge defined by [4]

$$Q_\zeta = \frac{1}{8\pi} \int_{\partial\Sigma} k_\zeta , \quad (3.10)$$

where  $\partial\Sigma$  is a spatial slice, and 2-form  $k_\zeta$  is defined as

$$k_\zeta[h, g] = \frac{1}{2} \left[ \zeta_\nu \nabla_\mu h - \zeta_\nu \nabla_\sigma h_\mu^\sigma + \zeta_\sigma \nabla_\nu h_\mu^\sigma + \frac{1}{2} h \nabla_\nu \zeta_\mu \right. \\ \left. - h_\nu^\sigma \nabla_\sigma \zeta_\mu + \frac{1}{2} h_{\nu\sigma} (\nabla_\mu \zeta^\sigma + \nabla^\sigma \zeta_\mu) \right] * (dx^\mu \wedge dx^\nu) , \quad (3.11)$$

where  $*$  denotes the Hodge dual.

The Dirac brackets of the conserved charges are just the common forms of the Virasoro algebras with a central term

$$\{Q_{\zeta_m}, Q_{\zeta_n}\}_{D.B.} = Q_{[\zeta_m, \zeta_n]} + \frac{1}{8\pi} \int_{\partial\Sigma} k_{\zeta_m} [\mathcal{L}_{\zeta_n} g, g] , \quad (3.12)$$

Translated into the quantum version, the Virasoro algebra is given by

$$[L_m, L_n] = (m - n) L_{m+n} + \frac{c}{12} (m^3 + \alpha m) \delta_{m+n, 0} , \quad (3.13)$$

where  $c$  denote the central charge corresponding to the diffeomorphism and  $\alpha$  is a trial constant. It follows that

$$\frac{1}{8\pi} \int_{\partial\Sigma} k_{\zeta_m} [\mathcal{L}_{\zeta_n} g, g] = -\frac{i}{12} c (m^3 + \alpha m) \delta_{m+n, 0} , \quad (3.14)$$

Evaluating the integral for the case of the near-horizon metric of extremal linear dilaton black hole, we find the central charge

$$c = 6r_0 a . \quad (3.15)$$

It should be noted that the central charge can also be written as  $c = 12J$ . This relation between the central charge and angular momentum is just of the same form as that for Kerr black hole in [1] and other examples of the Extremal Kerr/CFT dual.

After obtaining the central charge of the extremal linear dilaton black hole, we now begin to get its CFT entropy. To get this, we have to calculate the generalized temperature with respect to the Frolov-Thorne vacuum. We consider the quantum field with eigenmodes of the asymptotic energy  $\omega$  and angular momentum  $m$ , which are given by the following form

$$e^{-i\omega t + im\varphi} = e^{-i\left(\omega - \frac{m}{r_0}\right) \frac{\lambda}{\epsilon} \hat{t} + im\hat{\varphi}} = e^{-in_R \hat{t} + in_L \hat{\varphi}} , \quad (3.16)$$

with

$$n_R = \left(\omega - \frac{m}{r_0}\right) \frac{\lambda}{\epsilon} , \quad n_L = m . \quad (3.17)$$

The correspondence Boltzmann factor is of the form

$$e^{-\frac{\omega - m\Omega}{T_H}} = e^{-\frac{n_R}{T_R} - \frac{n_L}{T_L}} , \quad (3.18)$$

where the left and right temperatures are given by

$$\begin{aligned} T_R &= \frac{\lambda}{\epsilon} T_H , \\ T_L &= \frac{T_H}{\frac{1}{r_0} - \Omega_H} . \end{aligned} \quad (3.19)$$

In the extremal limit  $M \rightarrow a$ , the left and right temperatures reduce to

$$T_R = 0 , \quad T_L = \frac{1}{2\pi} . \quad (3.20)$$

According to the Cardy formula the entropy for a unitary CFT, we can obtain the microscopic entropy of the extremal linear dilaton black hole

$$S_{CFT} = \frac{\pi^2}{3} c T_L = \pi r_0 a = S_{BH}(T_H = 0) , \quad (3.21)$$

which precisely agrees with the Bekenstein-Hawking entropy. So one can conjecture that the extremal rotating linear dilaton black hole is dual to a two dimensional chiral conformal field theory.

#### 4. Hidden conformal symmetry of non-extremal linear dilaton black hole

In this section, we investigate the hidden conformal symmetry of the non-extremal rotating linear dilaton black hole. This hidden  $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$  conformal symmetry is not derived from the spacetime geometry itself, but can be probe by the perturbation field. Let us consider the wave equation of the neutral massless scalar field in the background of the non-extremal rotating linear dilaton black hole (2.2), which is given by the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) \Phi = 0 . \quad (4.1)$$

After performing the variable separation of scalar field  $\Phi = e^{-i\omega t} R(r) P(\theta) e^{im\varphi}$ , with  $\omega$  and  $m$  are quantum numbers, one can obtain the following two equations relative to radial part and angular part respectively

$$\partial_r (\Delta \partial_r) R(r) + \left( \frac{(r_0 r \omega - m a)^2}{\Delta} - \mathcal{K}^2 \right) R(r) = 0 , \quad (4.2)$$

$$\frac{1}{\sin\theta} \partial_\theta (\sin\theta \partial_\theta) P(\theta) - \left( \frac{m^2}{\sin^2\theta} - \mathcal{K}^2 \right) P(\theta) = 0 . \quad (4.3)$$

Unlike the case of Kerr black hole, now the angular equation is just that for the associated Legendre functions. So we have the separation constant

$$\mathcal{K}^2 = l(l+1) . \quad (4.4)$$



While the radial equation can be rewritten as

$$\begin{aligned} \partial_r(\Delta\partial_r)R(r) + \frac{(\omega r_0 r_+ - ma)^2}{(r - r_+)(r_+ - r_-)}R(r) - \frac{(\omega r_0 r_- - ma)^2}{(r - r_-)(r_+ - r_-)}R(r) \\ = (l(l+1) - \omega^2 r_0^2)R(r) \quad , \end{aligned} \quad (4.5)$$

which can be exactly solved by the hypergeometric function without taking the near-horizon limit as we will show in the following.

In order to solve the radial equation exactly, the new variable  $z$  should be introduced like this

$$z = \frac{r - r_+}{r - r_-} \quad . \quad (4.6)$$

Then the radial equation (4.5) can be expressed as the following after some algebra

$$z(1-z)\frac{d^2 R}{dz^2} + (1-z)\frac{dR}{dz} + \left(\frac{A}{z} + \frac{B}{1-z} + C\right)R = 0 \quad , \quad (4.7)$$

where  $A$ ,  $B$  and  $C$  are given by

$$\begin{aligned} A &= \frac{(\omega r_0 r_+ - ma)^2}{(r_+ - r_-)^2} \quad , \\ B &= \omega^2 r_0^2 - l(l+1) \quad , \\ C &= -\frac{(\omega r_0 r_- - ma)^2}{(r_+ - r_-)^2} \quad . \end{aligned} \quad (4.8)$$

By redefining the function  $R(z)$  as

$$R(z) = z^\alpha (1-z)^\beta F(z) \quad , \quad (4.9)$$

with

$$\begin{aligned} \alpha &= -i\sqrt{A} = -i\frac{(\omega r_0 r_+ - ma)}{r_+ - r_-} \quad , \\ \beta &= \frac{1}{2}(1 - \sqrt{1 - 4B}) = \frac{1}{2} - \sqrt{(l + \frac{1}{2})^2 - \omega^2 r_0^2} \quad , \end{aligned} \quad (4.10)$$

the equation (4.7) can be transformed into the standard hypergeometric function form

$$z(1-z)F'' + (c - (1+a+b)z)F' - abF = 0 \quad , \quad (4.11)$$

where the parameters  $a$ ,  $b$  and  $c$  are given by

$$\begin{aligned} a &= \alpha + \beta + i\sqrt{-C} = \frac{1}{2} - \sqrt{(l + \frac{1}{2})^2 - \omega^2 r_0^2} - i\omega r_0 \quad , \\ b &= \alpha + \beta - i\sqrt{-C} = \frac{1}{2} - \sqrt{(l + \frac{1}{2})^2 - \omega^2 r_0^2} - i\frac{\omega r_0(r_+ + r_-) - 2ma}{r_+ - r_-} \quad , \\ c &= 1 + 2\alpha = 1 - \frac{2i(\omega r_0 r_+ - ma)}{r_+ - r_-} \quad . \end{aligned} \quad (4.12)$$

The hypergeometric equation (4.11) has two linearly independent solutions which are given by

$$f_1 = F(a, b, c; z) \quad , \quad f_2 = z^{(1-c)} F(a - c + 1, b - c + 1, 2 - c; z) \quad , \quad (4.13)$$

where  $F(a, b, c; z)$  is just the so-called hypergeometric function. Then, it follows that the general solution of the radial equation (4.5) can be expressed as

$$R(z) = C_1 z^\alpha (1 - z)^\beta F(a, b, c; z) + C_2 z^{-\alpha} (1 - z)^\beta F(a - c + 1, b - c + 1, 2 - c; z) \quad (4.14)$$

It should be noted that, generally, the wave equation cannot be analytically solved and the solution must be obtained by matching solutions in an overlap region between the near-horizon and asymptotic regions. But, in the present case, we have obtained the general solution (4.14) of wave equation and shown that the radial equation (4.5) can be exactly solved by hypergeometric functions. As hypergeometric functions transform in representations of  $SL(2, \mathbb{R})$ , this suggests the existence of a hidden conformal symmetry. Now we will show that the radial equation can also be obtained by using of the  $SL(2, \mathbb{R})$  Casimir operator.

Introducing the coordinates

$$\begin{aligned} w^+ &= \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_R \varphi} \quad , \\ w^- &= \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_L \varphi + 2n_L t} \quad , \\ y &= \sqrt{\frac{r_+ - r_-}{r - r_-}} e^{\pi(T_L + T_R)\varphi + n_L t} \quad , \end{aligned} \quad (4.15)$$

with

$$T_R = \frac{r_+ - r_-}{4\pi a} \quad , \quad T_L = \frac{r_+ + r_-}{4\pi a} \quad , \quad n_L = \frac{1}{2r_0} \quad . \quad (4.16)$$

Then we can locally define the vector fields

$$\begin{aligned} H_1 &= i\partial_+ \quad , \\ H_0 &= i(w^+ \partial_+ + \frac{1}{2} y \partial_y) \quad , \\ H_{-1} &= i(w^{+2} \partial_+ + w^+ y \partial_y - y^2 \partial_-) \quad , \end{aligned} \quad (4.17)$$

and

$$\begin{aligned} \bar{H}_1 &= i\partial_- \quad , \\ \bar{H}_0 &= i(w^- \partial_- + \frac{1}{2} y \partial_y) \quad , \\ \bar{H}_{-1} &= i(w^{-2} \partial_- + w^- y \partial_y - y^2 \partial_+) \quad , \end{aligned} \quad (4.18)$$

These vector fields obey the  $SL(2, \mathbb{R})$  Lie algebra

$$[H_0, H_{\pm 1}] = \mp i H_{\pm 1} \quad , \quad [H_{-1}, H_1] = -2i H_0 \quad , \quad (4.19)$$

and similarly for  $(\bar{H}_0, \bar{H}_{\pm 1})$ . The  $\text{SL}(2, \mathbb{R})$  quadratic Casimir operator is

$$\begin{aligned}\mathcal{H}^2 = \bar{\mathcal{H}}^2 &= -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) \\ &= \frac{1}{4}(y^2 \partial_y^2 - y \partial_y) + y^2 \partial_+ \partial_- .\end{aligned}\tag{4.20}$$

In terms of the  $(t, r, \varphi)$  coordinates, the  $\text{SL}(2, \mathbb{R})$  generators are given by

$$\begin{aligned}H_1 &= ie^{-2\pi T_R \varphi} \left[ \sqrt{\Delta} \partial_r + \frac{1}{2\pi T_R} \frac{r-M}{\sqrt{\Delta}} \partial_\varphi - \frac{T_L}{T_R} \frac{r_0(Mr-a^2)}{M\sqrt{\Delta}} \partial_t \right] , \\ H_0 &= i \left[ \frac{1}{2\pi T_R} \partial_\varphi - \frac{T_L}{T_R} r_0 \partial_t \right] , \\ H_{-1} &= ie^{2\pi T_R \varphi} \left[ -\sqrt{\Delta} \partial_r + \frac{1}{2\pi T_R} \frac{r-M}{\sqrt{\Delta}} \partial_\varphi - \frac{T_L}{T_R} \frac{r_0(Mr-a^2)}{M\sqrt{\Delta}} \partial_t \right] ,\end{aligned}\tag{4.21}$$

and

$$\begin{aligned}\bar{H}_1 &= ie^{-(2\pi T_L \varphi + \frac{t}{r_0})} \left[ \sqrt{\Delta} \partial_r - \frac{1}{4\pi T_R} \frac{r_+ - r_-}{\sqrt{\Delta}} \partial_\varphi + \frac{r_0 r}{\sqrt{\Delta}} \partial_t \right] , \\ \bar{H}_0 &= -ir_0 \partial_t , \\ \bar{H}_{-1} &= ie^{2\pi T_L \varphi + \frac{t}{r_0}} \left[ -\sqrt{\Delta} \partial_r - \frac{1}{4\pi T_R} \frac{r_+ - r_-}{\sqrt{\Delta}} \partial_\varphi + \frac{r_0 r}{\sqrt{\Delta}} \partial_t \right] ,\end{aligned}\tag{4.22}$$

and the  $\text{SL}(2, \mathbb{R})$  quadratic Casimir operator becomes

$$\mathcal{H}^2 = \partial_r \Delta \partial_r - \frac{(r_0 r_+ \partial_t + a \partial_\varphi)^2}{(r - r_+)(r_+ - r_-)} + \frac{(r_0 r_- \partial_t + a \partial_\varphi)^2}{(r - r_-)(r_+ - r_-)} .\tag{4.23}$$

So for the scalar field with sufficient low energy  $\omega r_0 \ll 1$ , the near region wave equation can be written as

$$\mathcal{H}^2 \Phi = \bar{\mathcal{H}}^2 \Phi = l(l+1) \Phi ,\tag{4.24}$$

and the conformal weights of dual operator of the massless field  $\Phi$  should be

$$(h_L, h_R) = (l, l) .\tag{4.25}$$

So we have uncovered the hidden  $\text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R$  symmetry of the non-extremal rotating linear dilaton black hole. It should be noted that this symmetry is only locally defined and is spontaneously broken to  $\text{U}(1)_L \times \text{U}(1)_R$  symmetry because of the periodic identification in the  $\varphi$  coordinate. If one conjectures that the non-extremal rotating linear dilaton black hole is dual to a CFT, the broken of the conformal symmetry leads to the left temperature  $T_L$  and right temperature  $T_R$  of the dual conformal field.

As a check of this conjecture, we want to calculate the microscopic entropy of the dual CFT, and compare it with the Bekenstein-Hawking entropy of the non-extremal linear dilaton black hole. For the extremal case, the central charges can be derived from an analysis of the asymptotic symmetry group as we did in the last section. However, we did not know how to extend this calculation away from extremality. As did in [41], we will

simply assume that the conformal symmetry found here connects smoothly to that of the extreme limit and the central charge still keeps the same as the extremal case, which is given by Eq.(3.15). The microscopic entropy of the dual CFT can be computed by the Cardy formula

$$S_{CFT} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = \pi r_0 r_+ = S_{BH} , \quad (4.26)$$

which matches with the black hole Bekenstein-Hawking entropy.

## 5. Conclusion

In this paper, we have extend the recently proposed Kerr/CFT correspondence to examine the dual conformal field of the extremal and non-extremal rotating linear dilaton black holes respectively. Firstly, for the extremal black hole, we have obtained its near horizon geometry and calculated the central charge and temperature of the dual conformal field by employing the approach of GHSS. It is shown that the microscopic entropy calculated by using Cardy formula agrees with the Bekenstein-Hawking entropy of the extremal black hole. Then, for the non-extremal case, we have investigated the hidden conformal symmetry of linear dilaton black hole by studying the wave equation of a massless scalar field, and found the left and right temperatures of the proposed dual conformal field. Furthermore, it is checked that the entropy of non-extremal linear dilaton black hole can also be reproduced by using Cardy formula.

## Acknowledgement

RL would like to thank Ming-Fan Li for helpful discussions. The work of JRR was supported by the Cuiying Programme of Lanzhou University (225000-582404) and the Fundamental Research Fund for Physics and Mathematic of Lanzhou University(LZULL200911).

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